The Find Function

The function \texttt{Find} returns a solution to a system of equations given by a solve block. You can use \texttt{Find} to solve a linear system, as with \texttt{lsolve}, or to solve nonlinear systems.

The example below solves a system in the unknowns $\alpha$ and $\beta$:

$$
\alpha := 0 \quad \beta := 0
$$

These are initial guess values for $\alpha$ and $\beta$. The algorithm for \texttt{Find} starts at these values and moves toward a solution.

Given

$$\sqrt{\alpha} + \sin(\beta) = 1.5$$

$$\alpha + \beta = 3$$

\texttt{Find}(\alpha, \beta) = \begin{pmatrix} 0.637 \\ 2.363 \end{pmatrix}

\texttt{Find}(\alpha, \beta) \) gives a solution to the system.

$$\alpha = 0.637$$

$$\beta = 2.363$$

Note: The entries of the solution vector correspond to the variables in the same order that the variables appear after \texttt{Find}. In the previous example, \texttt{Find}(\beta, \alpha) returns the entries of the solution vector in reverse order.
To check a solution returned by \textbf{Find}, assign the results to variables.

\[
\alpha := 0 \quad \beta := 0
\]

Given

\[
\sqrt{\alpha} + \sin(\beta) = 1.5
\]

\[
\alpha + \beta = 3
\]

\[
\begin{pmatrix}
\alpha \\
\beta
\end{pmatrix} := \text{Find}(\alpha, \beta)
\]

You can use the same names for the results as for the unknown variables.

Evaluate the left hand sides of the system

\[
\sqrt{\alpha} + \sin(\beta) = 1.5
\]

\[
\alpha + \beta = 3
\]

to confirm that the solution is correct.

\textbf{Multiple Solutions}

Look at the system

\[
x := 1 \quad y := 1
\]

Given

\[
2x^2 + 3y^2 = 59
\]

\[
4y = x + 8
\]

\[
\text{Find}(x, y) = \begin{pmatrix} 4 \\ 3 \end{pmatrix}
\]

Explanation_of_Find
The first equation represents an ellipse, while the second represents a straight line. These are plotted below, along with the solution point.

\[ 2x^2 + 3y^2 = 59 \]
\[ 4y = x + 8 \]

The diagram shows the ellipse and the line intersecting at the point (4,3), indicating the solution block solution (4,3) is the solution to the system of equations.
As the graph shows, the solution corresponds to the point in the first quadrant where the curve and the line intersect. However, there is another solution to the system, corresponding to the point of intersection in the second quadrant. How can you get \textbf{Find} to return this second solution?

One way is by changing the guess values. Keep in mind that the result returned by the function \textbf{Find} (as well as the functions \textbf{Minerr}, \textbf{Minimize}, and \textbf{Maximize}) is directly related to the guess values for the unknown variables, and at most one solution is returned for a given set of guess values. So changing the guess values might lead to a different solution.

Looking at the graph above, you can see that the second solution lies in the second quadrant. So it seems reasonable to try guess values corresponding to a point - the guess point - that also lies in the second quadrant. Try the guess point (-3, 3).

\begin{align*}
x &:= -3 \\
y &:= 3
\end{align*}

Given

\begin{align*}
2x^2 + 3y^2 &= 59 \\
4y &= x + 8
\end{align*}

\textbf{Find}(x, y) = \begin{pmatrix} -5.371 \\ 0.657 \end{pmatrix}

This time \textbf{Find} returns the second solution.

Usually, if you choose a guess point close to a solution, \textbf{Find} returns that solution. However, as with the \textbf{root} function, \textbf{Find} does not always return the solution that is closest to the given guess point.
You can see the relationship between guess points and their corresponding solutions graphically by defining a function that takes a guess point to the resulting solution.

Given

\[2x^2 + 3y^2 = 59\]
\[4y = x + 8\]

\[Pt(x, y) := \text{Find}(x, y)\]

For any guess point \((x, y)\), the function \(Pt(x,y)\) returns one of the two solutions. For example:

\[Pt(-3, 3) = \begin{pmatrix} -5.371 \\ 0.657 \end{pmatrix}\]

Now, see what happens when you apply the \(Pt\) function to 25 guess points, equally spaced on a circle of radius 4 with center at the origin. Draw a line from each guess point to the solution produced by the \(Pt\) function for that guess. The resulting plot is quite interesting.

\[
\begin{align*}
R &:= 4 \\
N &:= 25 \\
i &:= 0 \ldots N - 1 \\
\begin{pmatrix} X_{0,i} \\ Y_{0,i} \end{pmatrix} &:= R \cdot \begin{pmatrix} \cos \left( i \cdot \frac{2 \cdot \pi}{N} \right) \\ \sin \left( i \cdot \frac{2 \cdot \pi}{N} \right) \end{pmatrix} \\
\begin{pmatrix} X_{1,i} \\ Y_{1,i} \end{pmatrix} &:= Pt(X_{0,i}, Y_{0,i})
\end{align*}
\]
Notice that most guess points in the right half-plane \((x > 0)\) lead to the solution \((4, 3)\). However, some points in the right half-plane lead to the solution \((-3.71, 0.657)\).

Try changing \(R\) to 6 in the example above to see what happens when the guess points lie on a circle of radius 6.

Note that the method for \textbf{Find} in this solve block has been set to Levenberg-Marquardt, a very stable routine that is tolerant of poor guesses. You can choose a different method by right-clicking the \textbf{Find} function, selecting \textbf{Nonlinear} from the drop-down menu, and selecting one of the choices. Different methods can lead to different solutions even with the same guess point.
Guess Point
\[ x = 1.236 \quad y = -3.804 \]

**Solution Using Levenberg-Marquardt Method**

Given
\[ 2x^2 + 3y^2 = 59 \]
\[ 4y = x + 8 \]

Find \( (x, y) = \left( -5.371, 0.657 \right) \)
Solution Using Conjugate Gradient Method

Given

\[ 2x^2 + 3y^2 = 59 \]

\[ 4y = x + 8 \]

Find \( (x, y) = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \)

What these examples show is that choosing guess values is actually a guessing game. A picture can help you identify the guess points that return the solutions you are looking for.

Errors and Problems with No Solutions

Sometimes there might be no solution, or Mathcad might not find a solution. In either case, Find displays the error message "No solution was found."

Here's an example of a problem with no solution

\[ u := 1 \quad v := 1 \]
Given

\[ u + v = 2 \]
\[ u + v = 3 \]

Find \( (u, v) = 1 \)

The problem asks for numbers \( u \) and \( v \) that add to both 2 and 3, which is impossible.
\textbf{Find} also returns this error message if there is a solution but the solver cannot find it. One example is

\[ z := 1 \]

Given

\[ \sin(z) = z^2 + 1 \]

\textbf{Find}(z) = \text{null}

The problem here is that the only solutions to the given equation are complex numbers. (Graph \( \sin(z) \) and \( z^2 + 1 \), and you'll see the curves do not intersect.) The real guess value of \( z := 1 \) sets the solver off in the wrong direction. In this case, just as with the root function, trying a complex guess may help

\[ z := 1 + i \]

Given

\[ \sin(z) = z^2 + 1 \]

\textbf{Find}(z) = 0.488 + 0.785i

\textbf{Find} returns an error if there are any missing guess values.

Given

\[ \frac{p^2}{1 + p} = \frac{1}{p} \]

\textbf{Find}(p) = \text{null}

The error message informs you that the variable is undefined.
\textbf{Find} also returns an error if any of the functions in the solve block is undefined at a guess value. For example:

\begin{align*}
  x := & -3 & y := & 4 \\
  \text{Given} & & & \\
  \Gamma(x) = & y + 1 & x + y = & 7 \\
  \text{Find}(x, y) = & \]

\textbf{Find} returns the error message "This value cannot be 0 or a negative integer." At first this is confusing. To find the source of the error, right-click \textbf{Find} and select \textbf{Trace Error}. Then click the \textbf{First} button in the \textbf{Trace Error} dialog. The cursor lands on the \textbf{Gamma} function, telling you that this is where the error occurs. The \textbf{Gamma} function is undefined at the value \(x := -3\).

\[ \Gamma(-3) = \]

Changing the value of \(x\) solves the problem.
Complex Solutions

Solve blocks sometimes return complex solutions even when the guess values are real. The following example, in which the solver method is set to Levenberg-Marquardt, illustrates this.

\[
\begin{align*}
\vec{u} := 1 & \quad \vec{v} := 2 \\
\text{Given} & \\
|\vec{u}| = 2 & \quad \frac{\sin(\vec{v})}{\vec{v} \cdot i} = \frac{\cos(\vec{u})}{\vec{u}} \\
\begin{pmatrix} \vec{u}s \\ \vec{v}s \end{pmatrix} := \text{Find}(\vec{u}, \vec{v}) \\
\vec{u}s = 1.814 + 0.843i & \quad \vec{v}s = 2.181 + 0.704i \\
|\vec{u}s| = 2 & \\
\frac{\sin(\vec{v}s)}{\vec{v}s \cdot i} = 0.044 - 0.486i & \quad \frac{\cos(\vec{u}s)}{\vec{u}s} = 0.044 - 0.486i
\end{align*}
\]

Try changing the guess values for this solve block from real to complex, and to different values to see how the results change.

When solve blocks begin solving a problem, they evaluate the constraints at the guess values as a check to see if the problem is real or complex. If the values of the constraints are complex at the guesses, the solve block can produce complex solutions even if the guess values themselves are real.

In other cases, where the constraints are only complex over a portion of their domain, you might be surprised by a complex result. This can occur if the solver, in the course of calculating its iterations, wanders into a complex region of solutions as it refines the guesses into solutions. Check the constraints in the regions of the guess value if you get real answers when you expect complex ones, or vice versa.