Comprehensive Mechanics Sample Problem

Given: 2-1/2” schedule 40 pipe made of Nylon 6/6

From the load as shown, is the pipe safe at point \( H \)?

Step 1: Resolve 50 lb force into \( x \) and \( z \) components
\[
F_x = (50 \text{ lb}) \cos 30^\circ = 43.3 \text{ lb} \\
F_z = (50 \text{ lb}) \sin 30^\circ = -25.0 \text{ lb}
\]

Step 2: Determine equivalent force and couple system @ point \( B \)
\[
F_x = 43.3 \text{ lb} \\
F_z = -25.0 \text{ lb} \\
M_x = T = F_z(10\text{ in}) = (-25.0 \text{ lb})(10 \text{ in}) = -250 \text{ lb-in} \\
M_z = F_x(10\text{ in}) = (43.3 \text{ lb})(10 \text{ in}) = -433 \text{ lb-in}
\]

Step 3: Determine equivalent force and couple system @ point \( C \)
\[
F_x = 43.3 \text{ lb} \\
F_z = -25.0 \text{ lb} \\
M_x = -250 \text{ lb-in} \\
M_z = -433 \text{ lb-in} \\
M_y = F_z(6\text{ in} + 4\text{ in}) = (-25.0 \text{ lb})(10 \text{ in}) = 250 \text{ lb-in}
\]

Step 4: Determine maximum radius @ point \( C \)
From ref. table: \( OD = 2.875'' \)
Let \( c = r_o = 1.4375 \text{ in} \)

Step 5: Determine area of cross-section @ point \( C \)
From ref. table: \( ID = 2.469'', \) let \( r_i = 1.2345 \text{ in} \)
\[
A = \pi(r_o^2 - r_i^2) = \pi[(1.4375 \text{ in})^2 - (1.2345 \text{ in})^2] = 1.704 \text{ in}^2
\]

Step 6: Determine second moment (moment of inertia) of cross-section at point \( C \)
\[
l = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}[(1.4375 \text{ in})^4 - (1.2345 \text{ in})^4] = 1.5296 \text{ in}^4
\]

Step 7: Determine polar moment of inertia of cross-section at point \( C \)
\[
J = \frac{\pi}{2}(r_o^4 - r_i^4) = \frac{\pi}{2}[(1.4375 \text{ in})^4 - (1.2345 \text{ in})^4] = 3.059 \text{ in}^4
\]

Step 8: Determine direct normal stress at point \( H \)
\[
\sigma = \frac{P}{A} = \frac{F_x}{A} = \frac{43.3 \text{ lb}}{1.704 \text{ in}^2} = 25.41 \text{ psi}
\]
Step 9: Determine bending normal stress @ point

\[ \sigma = \frac{Mc}{I} = \frac{M \cdot r_o}{I} = \frac{(43.3 \text{ lb} \cdot \text{in})(1.4375 \text{ in})}{(1.5296 \text{ in}^4)} = 406.9 \text{ psi} \]

Step 10: Combine normal stresses @ point

\[ \sigma = 25.4 \text{ psi} + 406.9 \text{ psi} = 432.3 \text{ psi} \]

Step 11: Determine torsional shear stress at point

\[ \tau = \frac{T_c}{J} = \frac{M \cdot r_o}{J} = \frac{(250 \text{ lb} \cdot \text{in})(1.4375 \text{ in})}{(3.059 \text{ in}^4)} = 117.5 \text{ psi} \]

Step 12: Determine statical moment @ point

\[ Q = A_p \bar{y} = \frac{\pi r^2}{2} \cdot \frac{4r^3}{3} = \frac{2r^3}{3} (r_o^3 - r_i^3) = \frac{2}{3} [(1.4375 \text{ in})^3 - (1.2345 \text{ in})^3] = 0.726 \text{ in}^3 \]

Step 13: Determine transverse shear stress @ point

\[ \tau = \frac{VQ}{It} = \frac{F_c Q}{It} = \frac{(25.0 \text{ lb})(0.726 \text{ in}^3)}{(1.5296 \text{ in}^4)2(1.4375 \text{ in} - 1.2345 \text{ in})} = 29.2 \text{ psi} \]

Step 14: Combine shear stresses @ point

\[ \tau_{xz} = 117.5 \text{ psi} + 29.2 \text{ psi} = 146.7 \text{ psi} \]

Step 15: Determine principle stress 1 @ point

\[ \sigma_1 = \frac{\sigma_x + \sigma_z}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \frac{432.3}{2} + \sqrt{\left(\frac{432.3}{2}\right)^{2} + (146.7)^2} = 477.3 \text{ psi} \]

Step 16: Determine principle stress 2 @ point

\[ \sigma_2 = \frac{\sigma_x + \sigma_z}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \frac{432.3}{2} - \sqrt{\left(\frac{432.3}{2}\right)^{2} + (146.7)^2} = -45.0 \text{ psi} \]

Step 17: Determine equivalent (von Mises) stress @ point

\[ \sigma_{eq} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} = \sqrt{(477.3)^2 + (-45.0)^2 - (477.3)(-45.0)} = 501.3 \text{ psi} \]

Step 18: Determine allowable stress for Nylon 6/6 in flexure

From ref. table: \( s_y = 1000 \text{ psi} \)

Step 19: Compare equivalent stress to allowable stress

\[ N = \frac{s_y}{\sigma_{eq}} = \frac{1000 \text{ psi}}{501.3 \text{ psi}} = 1.995; \text{ N > 1 (OK, it is safe)} \]

NEXT: USE MOHR’S CIRCLE FOR FINDING PRINCIPAL STRESSES!!