The Find Function

The function \texttt{Find} returns a solution to a system of equations given by a \texttt{solve block}. You can use \texttt{Find} to solve a linear system, as with \texttt{lsolve}, or to solve nonlinear systems.

The example below solves a system in the unknowns $\alpha$ and $\beta$:

$$\alpha := 0 \quad \beta := 0$$

These are initial guess values for $\alpha$ and $\beta$. The algorithm for \texttt{Find} starts at these values and moves toward a solution.

Given

$$\sqrt{\alpha} + \sin(\beta) = 1.5$$

$$\alpha + \beta = 3$$

\texttt{Find}(\alpha, \beta) = \begin{pmatrix} 0.637 \\ 2.363 \end{pmatrix}$$

\texttt{Find}(\alpha, \beta) gives a solution to the system.

$$\alpha = 0.637$$

$$\beta = 2.363$$

Note: The entries of the solution vector correspond to the variables in the same order that the variables appear after \texttt{Find}. In the previous example, \texttt{Find}(\beta, \alpha) returns the entries of the solution vector in reverse order.
To check a solution returned by **Find**, assign the results to variables.

\[ \alpha := 0 \quad \beta := 0 \]

Given

\[ \sqrt{\alpha + \sin(\beta)} = 1.5 \]
\[ \alpha + \beta = 3 \]

\[ \begin{pmatrix} \alpha \\ \beta \end{pmatrix} := \text{Find}(\alpha, \beta) \]

You can use the same names for the results as for the unknown variables.

Evaluate the left hand sides of the system

\[ \sqrt{\alpha + \sin(\beta)} = 1.5 \]
\[ \alpha + \beta = 3 \]

to confirm that the solution is correct.

**Multiple Solutions**

Look at the system

\[ x := 1 \quad y := 1 \]

Given

\[ 2x^2 + 3y^2 = 59 \]
\[ 4y = x + 8 \]

\[ \text{Find}(x, y) = \begin{pmatrix} 4 \\ 3 \end{pmatrix} \]
The first equation represents an ellipse, while the second represents a straight line. These are plotted below, along with the solution point.

\[ 2x^2 + 3y^2 = 59 \]
\[ 4y = x + 8 \]

Solve block solution (4,3)
As the graph shows, the solution corresponds to the point in the first quadrant where the curve and the line intersect. However, there is another solution to the system, corresponding to the point of intersection in the second quadrant. How can you get \textbf{Find} to return this second solution?

One way is by changing the guess values. Keep in mind that the result returned by the function \textbf{Find} (as well as the functions \textbf{Minerr}, \textbf{Minimize}, and \textbf{Maximize}) is directly related to the guess values for the unknown variables, and at most one solution is returned for a given set of guess values. So changing the guess values might lead to a different solution.

Looking at the graph above, you can see that the second solution lies in the second quadrant. So it seems reasonable to try guess values corresponding to a point - the guess point - that also lies in the second quadrant. Try the guess point (-3, 3).

\[
x := -3 \quad y := 3
\]

Given

\[
2x^2 + 3y^2 = 59
\]

\[
4y = x + 8
\]

\[
\text{Find}(x, y) = \begin{pmatrix} -5.371 \\ 0.657 \end{pmatrix}
\]

This time \textbf{Find} returns the second solution.

Usually, if you choose a guess point close to a solution, \textbf{Find} returns that solution. However, as with the \textbf{root} function, \textbf{Find} does not always return the solution that is closest to the given guess point.