Coordinate Method:

The area of any polygon (regular or irregular – see Figure 1) can be determined using a method called the "Coordinate method." In this method, the coordinates of the vertices are systematically multiplied, then added or subtracted to obtain the area bounded by the polygon.

METHOD:

1. List the x and y-coordinates of each of the vertices in a column, repeating the starting point coordinate at the end of the list. The coordinates have to be listed in the order they occur while tracing around the polygon in either the clockwise or counterclockwise direction.

2. Multiply each x value by the next y-value in the list and add the products.

\[ \sum x_i(y_{i+1}) \]

3. Multiply each y-value by the next x-value in the list and add the products.

\[ \sum y_i(x_{i+1}) \]

4. Obtain the area by evaluating the expression:

\[ \text{Area} = \frac{1}{2} \left| \sum x_i(y_{i+1}) - \sum y_i(x_{i+1}) \right| \]

EXAMPLE: Find the area bounded by a polygon having 5 sides and coordinates \((X_1, Y_1)\) through \((X_5, Y_5)\).

<table>
<thead>
<tr>
<th>List of Coordinates</th>
<th>First Products</th>
<th>Second Products</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>Y</td>
<td>(X_i(Y_{i+1}))</td>
</tr>
<tr>
<td>X_1</td>
<td>Y_1</td>
<td>(X_1Y_2)</td>
</tr>
<tr>
<td>X_2</td>
<td>Y_2</td>
<td>(X_2Y_3)</td>
</tr>
<tr>
<td>X_3</td>
<td>Y_3</td>
<td>(X_3Y_4)</td>
</tr>
<tr>
<td>X_4</td>
<td>Y_4</td>
<td>(X_4Y_5)</td>
</tr>
<tr>
<td>X_5</td>
<td>Y_5</td>
<td>(X_5Y_6)</td>
</tr>
<tr>
<td>X_6 = X_1</td>
<td>Y_6 = Y_1</td>
<td></td>
</tr>
</tbody>
</table>

Sum = \(X_1Y_2 + X_2Y_3 + X_3Y_4 + X_4Y_5 + X_5Y_6\)

Area = \((1/2) \left| X_1Y_2 + X_2Y_3 + X_3Y_4 + X_4Y_5 + X_5Y_6 - (Y_1X_2 + Y_2X_3 + Y_3X_4 + Y_4X_5 + Y_5X_6) \right|\)

This method truly lends itself to a solution using a spreadsheet.
EXAMPLE: Find the area bounded by the polygon shown in the figure below using method of Area by Coordinates.

\[ \text{Area} = \frac{1}{2} | 76 - 118 | = \frac{1}{2} | -42 | = 21 \]

CHECK: Divide area into shapes having well-known area formulas:

\[ A = 9 + 1 + 6 + 2 + 1 + 2 = 21 \]

Checks